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AGS Division Technical Note
No. 204

CALCULATION OF MOTOR SPEED OF THE SIEMENS M.G. SET DURING A PULSE

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The motor generator system has been designed so that under normal circumstances the motor speed over the duration of an AGS magnet pulse varies by less than $\pm 2\%$. This is with a motor power input variation of less than ± 200 KW. Considering that the main magnet power supply output goes from +70 MW to -70 MW during the pulse, these power swings must be absorbed by the rotational energy of the MG set. This has been accomplished by designing the generator rotor with a large radius resulting in a large moment of inertia. At 1200 RPM, the stored energy is 310 MJ.

The motor speed at any time during a pulse can be determined by considering the electrical and mechanical relationships

$$1) \quad P_{in} = JW \frac{\partial W}{\partial t} + P_W + P_E$$

where

P_{in} = power into the motor

J = MG set moment of inertia $[40(10^3) \text{ kg M}^2]$

W = rotational speed in radians/sec

P_W = windage and bearing losses (850 KW)

P_E = electrical power

Three types of elements contribute to the electrical power term (P_E). They are resistance (P_R), inductance (P_L) and arc drop (P_A). Let us discuss these items a little further.

The arc drop term is attributed to the arc drop across the excitrons. For each excitron this is equivalent to a 16 volt battery in series 13 m Ω . For the whole power supply, this is equivalent to 128 volts in series with 50 m Ω . For convenience, this 50 m Ω and other power supply resistances are lumped in with the load resistance of 0.25 Ω to comprise a total resistance of 0.3 Ω . This 0.3 Ω will be used for the determination of the " P_R " term. This 128 volts will be used for the " P_A " term.

Now, the inductive term is something somewhat special. The main reason is that the prime source of the system inductance is that of the magnet whose inductance varies with the current. This variation is from 0.5 to 0.8 Henries. The main value used in this note will be 0.7 H. This includes also the reactive drop in the transformer connections, etc. It should also be pointed out that the " P_L " term can be either positive or negative depending upon whether the current is increasing or decreasing.

In light of these comments, we can then conclude that

$$2) \quad W \frac{\partial W}{\partial t} = \frac{(P_{in} - P_W)}{J} - \frac{(P_E + P_R + P_L)}{J}$$

Integrating both sides from 0 to t yields

$$3) \quad W^2 - W_0^2 = \left(\frac{P_{in} - P_W}{J/2} \right) t - \frac{1}{J/2} \int_0^t (P_E + P_R + P_L) dt$$

$$4) \quad W^2 = W_o^2 + \left(\frac{P_{in} - P_W}{J/2} \right) t - \frac{1}{J/2} \int_0^t (E_E I + RI^2 + LI \frac{dI}{dt}) dt$$

If "I" has form

$$5) \quad I = I_o + \alpha t$$

then,

$$6) \quad \frac{dI}{dt} = \alpha$$

and then

$$7) \quad W^2 = W_o^2 + \left(\frac{P_{in} - P_W}{J/2} \right) t - \frac{1}{J/2} \int_0^t [(EI_o + \alpha LI_o + RI_o^2) + (\alpha E_E + L\alpha^2 + 2\alpha RI_o)t + \alpha^2 Rt^2] dt$$

and

$$8) \quad W^2 = W_o^2 + \left(\frac{P_{in} - P_W}{J/2} \right) t - \left(\frac{E_E I_o + L\alpha I_o + RI_o^2}{J/2} \right) t - \left(\frac{E_E \alpha + 2R\alpha I_o + \alpha^2 L}{2(J/2)} \right) t^2 - \frac{\alpha^2 Rt^3}{3(J/2)}$$

During a pulse, four current segments can be identified. They are: injection porch, rectify, flattop, and invert. The current characteristics during each of these intervals are as follows:

Injection Porch

$$9) \quad I_{IP} = 2956 t \quad \left| \begin{array}{l} t \text{ 0.11 sec} \\ 0 \end{array} \right.$$

Rectify

$$10) \quad I_R = 325.16 + 10,500 t \quad \left| \begin{array}{l} t_R \\ 0 \end{array} \right.$$

Flattop

$$11) \quad I_F = I_F \text{ (constant)}$$

Invert

$$12) \quad I_I = I_F - 11,250 t \quad \left| \begin{array}{l} t_I=0 \\ 0 \end{array} \right.$$

Therefore, during the injection porch,

$$13) \quad W^2 = W_o^2 + \left(\frac{P_{in} - 0.85(10^6)}{2(10^4)} \right) t - 162.4t^2 - 43.69t^3$$

Where W_o is approximately 1.5% above the synchronous (rotational) speed. The synchronous speed is 1200 RPM or 125.66 radians per second. 1.5% above this is about 128 radians per second. W_o^2 is then 16,384.

During rectify we get

$$14) \quad W_R^2 = W_{oR}^2 + \left(\frac{P_{in} - 0.85(10^6)}{2(10^4)} \right) t_R - 121t_R - 2014t_R^2 - 551t_R^3$$

Where t_R is reckoned from the end of the front porch ($t = 0.11$) and W_{oR} is the speed at the end of the front porch.

During flattop

$$15) \quad W_F^2 = W_{oF}^2 + \left(\frac{P_{in} - 0.85(10^6)}{2(10^4)} \right) t_F - \left(\frac{128 I_{Fo} + 0.3 I_{Fo}^2}{2(10^4)} \right) t_F$$

Where t_F is reckoned from the beginning of flattop and I_{Fo} is the current at the beginning of flattop.

For invert, the following relationship exists

$$16) \quad W_I^2 = W_{oI}^2 + \left(\frac{P_{in} - 0.85(10^6)}{2(10^4)} \right) t_1 + \left(\frac{7747 I_{o1} - 0.3 I_{o1}^2}{2(10^4)} \right) t_1 - (2179 - 0.1688 I_{o1}) t_1^2 - 632.8 t_1^3$$

W_{o1} is the speed at beginning of invert at t_1 is reckoned from beginning of invert.

Assuming an M.G. set initial speed of 128 radians per second, the speed as a function of time has been calculated over the duration of a typical 28 GeV/c AGS magnet current pulse. This corresponds to a current of 5050 amperes. This was done for a 1.0 second and a 2.0 second flat-top. Two sets of calculations were made; one set for a motor net power input of four megawatts, the other for seven.

The results of these calculations are tabulated on the following pages. A plot is also included.

mvh

Distribution: Dept. S&P
Siemens Techs

INJECTION PORCH

P_{IN} - P_W = 4 MW

7 MW

(7)

0.01	0.01	SEC
29.58	29.56	AMPS
128.0077487	128.0136075	RADS
0.02	0.02	SEC
59.12	59.12	AMPS
128.015368	128.0270858	RADS
0.03	0.03	SEC
88.68	88.68	AMPS
128.0228599	128.0404337	RADS
0.04	0.04	SEC
118.24	118.24	AMPS
128.0302205	128.0536503	RADS
0.05	0.05	SEC
147.8	147.8	AMPS
128.0374498	128.0667347	RADS
0.06	0.06	SEC
177.36	177.36	AMPS
128.0445466	128.0796858	RADS
0.07	0.07	SEC
206.92	206.92	AMPS
128.0515102	128.0925027	RADS
0.08	0.08	SEC
236.48	236.48	AMPS
128.0583397	128.1051844	RADS
0.09	0.09	SEC
266.04	266.04	AMPS
128.0650331	128.1177299	RADS
0.1	0.1	SEC
295.6	295.6	AMPS
128.0715906	128.1301382	RADS
0.11	0.11	SEC
325.16	325.16	AMPS
128.0780106	128.1424083	RADS
0.12	0.12	SEC
354.72	354.72	AMPS
128.0842920	128.1545393	RADS

PIN-PL

4mw

7mw

4mw

7mw

0.01
430.16
128.0865878

0.01
430.16
128.1626856

SEC
AMPS
RADS

0.02
535.16
128.0872981

0.02
535.16
128.1692472

SEC
AMPS
RADS

✓ 0.03
640.16
128.0864101

✓ 0.03
640.16
128.1742114

SEC
AMPS
RADS

0.04
745.16
128.0839111

0.04
745.16
128.1775654

SEC
AMPS
RADS

✓ 0.05
850.16
128.0797879

✓ 0.05
850.16
128.1792966

SEC
AMPS
RADS

0.06
955.16
128.0740275

0.06
955.16
128.179392

SEC
AMPS
RADS

0.07
1060.16
128.0666168

0.07
1060.16
128.1778388

SEC
AMPS
RADS

✓ 0.08
1165.16
128.0575426

✓ 0.08
1165.16
128.174624

SEC
AMPS
RADS

0.09
1270.16
128.0467917

0.09
1270.16
128.1697346

SEC
AMPS
RADS

0.1
1375.16
128.0343506

0.1
1375.16
128.1631575

SEC
AMPS
RADS

0.11
1480.16
128.020206

0.11
1480.16
128.1548796

SEC
AMPS
RADS

0.12
1585.16
128.0043445

0.12
1585.16
128.1448876

SEC
AMPS
RADS

✓ 0.13
1690.16
127.9867524

0.14
1795.16
127.9674161

0.15
1900.16
127.946322

0.16
2005.16
127.9234562

0.17
2110.16
127.8988048

✓ 0.18
2215.16
127.8723539

0.19
2320.16
127.8440895

0.2
2425.16
127.8139974

0.21
2530.16
127.7820634

0.22
2635.16
127.7482731

✓ 0.23
2740.16
127.7126122

0.24
2845.16
127.6750661

0.25
2950.16
127.6356203

✓ 0.13
1690.16
128.1331682

0.14
1795.16
128.1197081

0.15
1900.16
128.1044937

0.16
2005.16
128.0875117

0.17
2110.16
128.0687483

✓ 0.18
2215.16
128.0481898

0.19
2320.16
128.0258225

0.2
2425.16
128.0016326

0.21
2530.16
127.975606

0.22
2635.16
127.9477288

✓ 0.23
2740.16
127.9179867

0.24
2845.16
127.8863657

0.25
2950.16
127.8528512

SEC
AMPS
RADS

SEC
AMPS
RADS

SEC
AMPS
RADS

SEC
AMPS
RADS

SEC
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SEC
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SEC
AMPS
RADS

SEC
AMPS
RADS

SEC
AMPS
RADS

SEC
AMPS
RADS

Pw-Pw

4MW

7MW

4MW

7MW

0.26 3055.16 127.5942599	0.26 3055.16 127.817429	SEC AMPS RADS	0.37 4210.16 127.0086199	0.37 4210.16 127.2976415	SEC AMPS RADS
0.27 3160.16 127.5509702	0.27 3160.16 127.7800846	SEC AMPS RADS	✓ 0.38 4315.16 126.9430969	✓ 0.38 4315.16 127.238162	SEC AMPS RADS
✓ 0.28 3265.16 127.5057363	✓ 0.28 3265.16 127.7408032	SEC AMPS RADS	0.39 4420.16 126.8754577	0.39 4420.16 127.1765771	SEC AMPS RADS
0.29 3370.16 127.458543	0.29 3370.16 127.6995701	SEC AMPS RADS	0.4 4525.16 126.8056857	0.4 4525.16 127.1128709	SEC AMPS RADS
0.3 3475.16 127.4093754	0.3 3475.16 127.6563706	SEC AMPS RADS	0.41 4630.16 126.7337645	0.41 4630.16 127.047027	SEC AMPS RADS
0.31 3580.16 127.358218	0.31 3580.16 127.6111896	SEC AMPS RADS	0.42 4735.16 126.6596773	0.42 4735.16 126.9790292	SEC AMPS RADS
0.32 3685.16 127.3050555	0.32 3685.16 127.5640121	SEC AMPS RADS	✓ 0.43 4840.16 126.5834072	✓ 0.43 4840.16 126.9088609	SEC AMPS RADS
✓ 0.33 3790.16 127.2498725	✓ 0.33 3790.16 127.5148229	SEC AMPS RADS	0.44 4945.16 126.5049373	0.44 4945.16 126.8365056	SEC AMPS RADS
0.34 3895.16 127.1926532	0.34 3895.16 127.4636067	SEC AMPS RADS	✓ 0.45 5050.16 126.4242503	✓ 0.45 5050.16 126.7619465	SEC AMPS RADS
0.35 4000.16 127.133382	0.35 4000.16 127.4103482	SEC AMPS RADS	0.46 5155.16 126.3413289	0.46 5155.16 126.6851665	SEC AMPS RADS
0.36 4105.16 127.0720429	0.36 4105.16 127.3550317	SEC AMPS RADS			

4MW

7MW

PIN-PW

4MW

7MW

0.03	0.03	SEC
126.3987523	126.7542686	RADS
0.13	0.13	SEC
126.3137218	126.7286721	RADS
0.23	0.23	SEC
126.2286339	126.7030704	RADS
0.33	0.33	SEC
126.1434887	126.6774635	RADS
0.43	0.43	SEC
126.058286	126.6518515	RADS
0.53	0.53	SEC
125.9730257	126.6262343	RADS
0.63	0.63	SEC
125.8877076	126.6006119	RADS
0.73	0.73	SEC
125.8023316	126.5749843	RADS
0.83	0.83	SEC
125.7168977	126.5493516	RADS
0.93	0.93	SEC
125.6314057	126.5237136	RADS
1.03	1.03	SEC
125.5458554	126.4980705	RADS
1.13	1.13	SEC
125.4602469	126.4724221	RADS

1.23	1.23	SEC
125.3745798	126.4467686	RADS
1.33	1.33	SEC
125.2888542	126.4211098	RADS
1.43	1.43	SEC
125.2030699	126.3954458	RADS
1.53	1.53	SEC
125.1172268	126.3697767	RADS
1.63	1.63	SEC
125.0313247	126.3441023	RADS
1.73	1.73	SEC
124.9453636	126.3184226	RADS
1.83	1.83	SEC
124.8593433	126.2927378	RADS
1.93	1.93	SEC
124.7732637	126.2670477	RADS
2.03	2.03	SEC
124.6871246	126.2413525	RADS

4MW	7MW	(P ₁₄ -P ₀)	4MW	7MW	
0.01 4937.66 125.6159413	0.01 4937.66 126.5735546	SEC AMPS RADS	0.13 3587.66 126.3666301	0.13 3587.66 127.3892666	SEC AMPS RADS
0.02 4825.16 125.6849175	0.02 4825.16 126.6479314	SEC AMPS RADS	0.14 3475.16 126.4212563	0.14 3475.16 127.4493392	SEC AMPS RADS
0.03 4712.66 125.7527709	0.03 4712.66 126.7211878	SEC AMPS RADS	0.15 3362.66 126.4746001	0.15 3362.66 127.5081351	SEC AMPS RADS
0.04 4600.16 125.8194882	0.04 4600.16 126.7933107	SEC AMPS RADS	0.16 3250.16 126.5266483	0.16 3250.16 127.565641	SEC AMPS RADS
0.05 4487.66 125.8850562	0.05 4487.66 126.8642873	SEC AMPS RADS	0.17 3137.66 126.5773874	0.17 3137.66 127.6218439	SEC AMPS RADS
0.06 4375.16 125.9494615	0.06 4375.16 126.9341045	SEC AMPS RADS	0.18 3025.16 126.6268041	0.18 3025.16 127.6767306	SEC AMPS RADS
0.07 4262.66 126.0126909	0.07 4262.66 127.0027492	SEC AMPS RADS	0.19 2912.66 126.6748848	0.19 2912.66 127.730288	SEC AMPS RADS
0.08 4150.16 126.0747311	0.08 4150.16 127.0702084	SEC AMPS RADS	0.2 2800.16 126.7216163	0.2 2800.16 127.782503	SEC AMPS RADS
0.09 4037.66 126.135569	0.09 4037.66 127.1364692	SEC AMPS RADS	0.21 2687.66 126.766985	0.21 2687.66 127.8333623	SEC AMPS RADS
0.1 3925.16 126.1951911	0.1 3925.16 127.2015184	SEC AMPS RADS	0.22 2575.16 126.8109774	0.22 2575.16 127.8828527	SEC AMPS RADS
0.11 3812.66 126.2535842	0.11 3812.66 127.2653431	SEC AMPS RADS	0.23 2462.66 126.85358	0.23 2462.66 127.930961	SEC AMPS RADS
0.12 3700.16 126.310735	0.12 3700.16 127.3279302	SEC AMPS RADS			

INVERT AFTER 1SEC FLAT-TOP CONTD

(12)

4MW	7MW	(P _{IN} -P _N)	4MW	7MW
0.24	0.24	SEC	0.35	0.35
2350.16	2350.16	AMPS	1112.66	1112.66
126.8947792	127.9776739	RADS	127.2514468	128.3956026
0.25	0.25	SEC	0.36	0.36
2237.66	2237.66	AMPS	1000.16	1000.16
126.9345616	128.0229783	RADS	127.274738	128.4245263
0.26	0.26	SEC	0.37	0.37
2125.16	2125.16	AMPS	887.66	887.66
126.9729135	128.0668606	RADS	127.2964466	128.4518795
0.27	0.27	SEC	0.38	0.38
2012.66	2012.66	AMPS	775.16	775.16
127.0098212	128.1093077	RADS	127.3165585	128.4776483
0.28	0.28	SEC	0.39	0.39
1900.16	1900.16	AMPS	662.66	662.66
127.0452712	128.1503061	RADS	127.3350598	128.501819
0.29	0.29	SEC	0.4	0.4
1787.66	1787.66	AMPS	550.16	550.16
127.0792496	128.1898425	RADS	127.3519361	128.5243777
0.3	0.3	SEC	0.41	0.41
1675.16	1675.16	AMPS	437.66	437.66
127.1117428	128.2279033	RADS	127.3671732	128.5453105
0.31	0.31	SEC	0.42	0.42
1562.66	1562.66	AMPS	325.16	325.16
127.142737	128.2644752	RADS	127.3807569	128.5646034
0.32	0.32	SEC	0.43	0.43
1450.16	1450.16	AMPS	212.66	212.66
127.1722184	128.2995447	RADS	127.3926727	128.5822425
0.33	0.33	SEC	0.44	0.44
1337.66	1337.66	AMPS	100.16	100.16
127.2001731	128.3330981	RADS	127.4029063	128.5982137
0.34	0.34	SEC	0.45	0.45
1225.16	1225.16	AMPS	-12.34	-12.34
127.2265872	128.3651219	RADS	127.4114432	128.6125029

4MW

7MW

4MW

7MW

0.01	0.01	SEC	0.12	0.12	SEC
4937.66	4937.66	AMPS	3700.16	3700.16	AMPS
124.7576929	126.31699	RADS	125.4572397	127.0728887	RADS
0.02	0.02	SEC	0.13	0.13	SEC
4825.16	4825.16	AMPS	3587.66	3587.66	AMPS
124.8271434	126.3915178	RADS	125.5135149	127.1343481	RADS
0.03	0.03	SEC	0.14	0.14	SEC
4712.66	4712.66	AMPS	3475.16	3475.16	AMPS
124.8954628	126.4649227	RADS	125.5685123	127.1945412	RADS
0.04	0.04	SEC	0.15	0.15	SEC
4600.16	4600.16	AMPS	3362.66	3362.66	AMPS
124.9626378	126.5371917	RADS	125.6222182	127.2534548	RADS
0.05	0.05	SEC	0.16	0.16	SEC
4487.66	4487.66	AMPS	3250.16	3250.16	AMPS
125.0286551	126.6083119	RADS	125.6746194	127.3110757	RADS
0.06	0.06	SEC	0.17	0.17	SEC
4375.16	4375.16	AMPS	3137.66	3137.66	AMPS
125.0935013	126.6782701	RADS	125.7257024	127.3673909	RADS
0.07	0.07	SEC	0.18	0.18	SEC
4262.66	4262.66	AMPS	3025.16	3025.16	AMPS
125.1571632	126.7470534	RADS	125.7754536	127.4223873	RADS
0.08	0.08	SEC	0.19	0.19	SEC
4150.16	4150.16	AMPS	2912.66	2912.66	AMPS
125.2196273	126.8146486	RADS	125.8238597	127.4760516	RADS
0.09	0.09	SEC	0.2	0.2	SEC
4037.66	4037.66	AMPS	2800.16	2800.16	AMPS
125.2808804	126.8810428	RADS	125.8709072	127.5283706	RADS
0.1	0.1	SEC	0.21	0.21	SEC
3925.16	3925.16	AMPS	2687.66	2687.66	AMPS
125.3409091	126.946223	RADS	125.9165823	127.5793312	RADS
0.11	0.11	SEC	0.22	0.22	SEC
3812.66	3812.66	AMPS	2575.16	2575.16	AMPS
125.3997	127.0101759	RADS	125.9608717	127.6289201	RADS

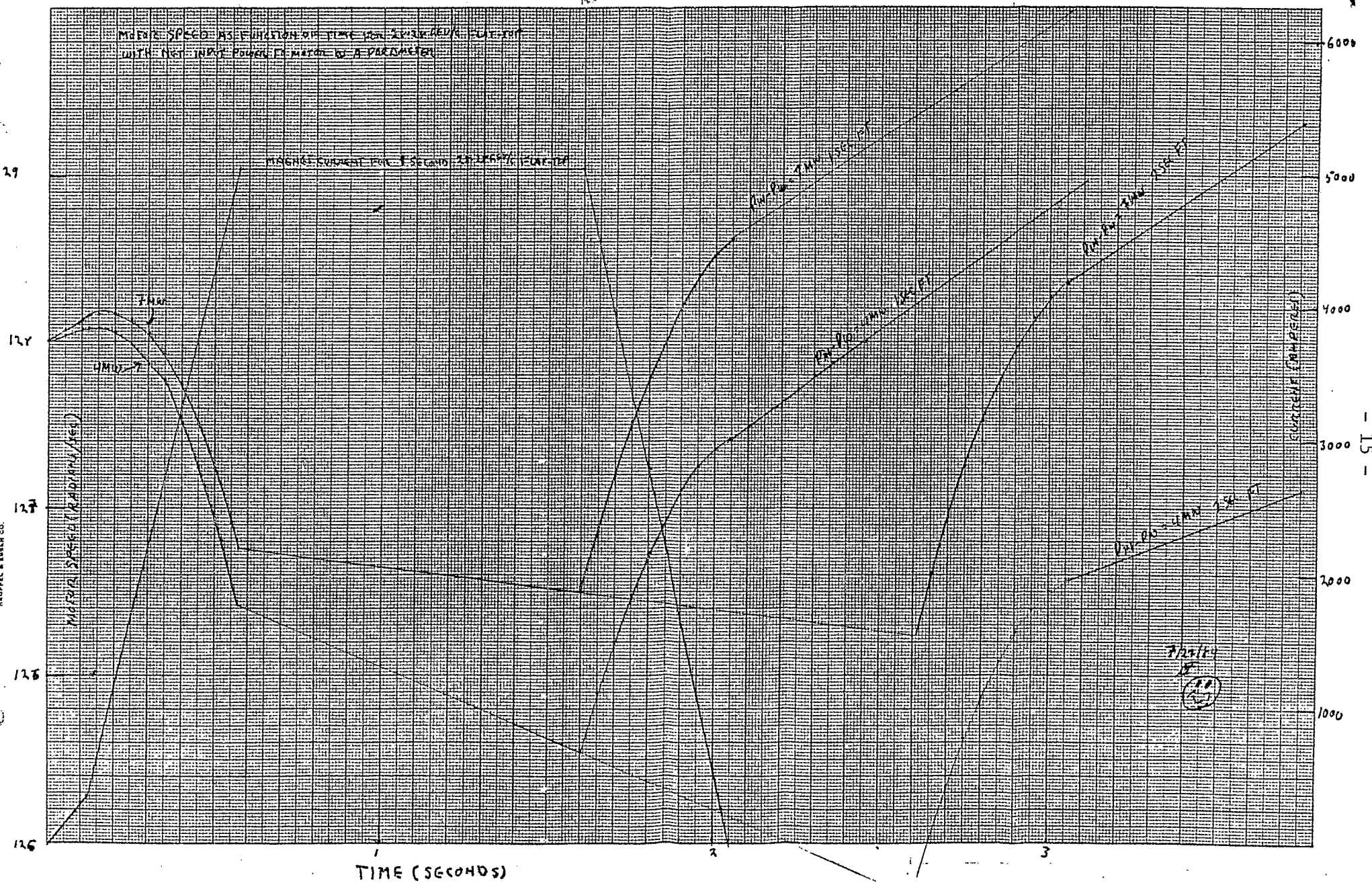
4MW

7MW

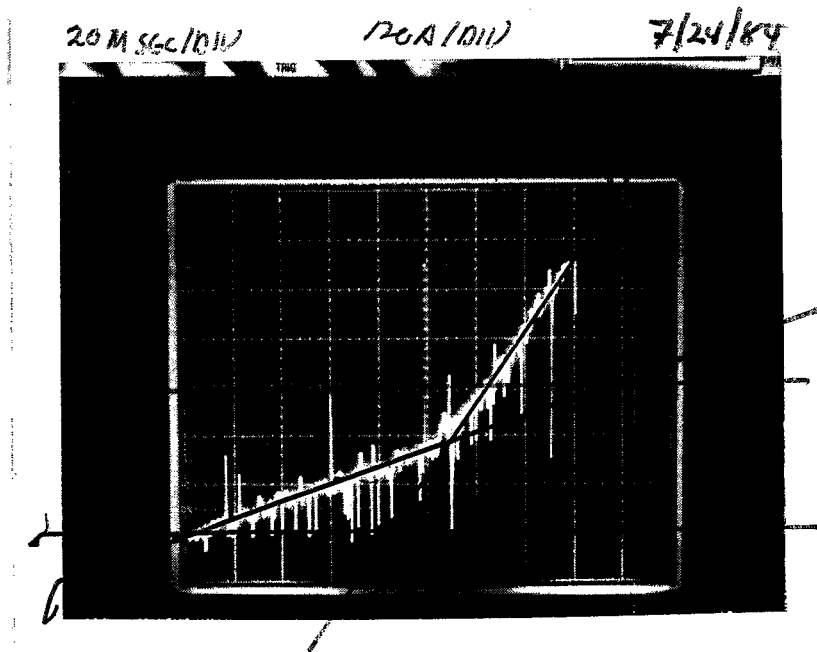
4MW

7MW

0.23	0.23	SEC	0.35	SEC
2462.66	2462.66	AMPS	1112.66	AMPS
126.0037618	127.6771241	RADS	126.4043036	128.1426861
0.24	0.24	SEC	0.36	SEC
2350.16	2350.16	AMPS	1000.16	AMPS
126.0452388	127.7239299	RADS	126.4277507	128.1716669
0.25	0.25	SEC	0.37	SEC
2237.66	2237.66	AMPS	887.66	AMPS
126.0852892	127.7693242	RADS	126.4496047	128.199074
0.26	0.26	SEC	0.38	SEC
2125.16	2125.16	AMPS	775.16	AMPS
126.1238994	127.8132936	RADS	126.4698514	128.2248937
0.27	0.27	SEC	0.39	SEC
2012.66	2012.66	AMPS	662.66	AMPS
126.1610555	127.8558249	RADS	126.4884765	128.249112
0.28	0.28	SEC	0.4	SEC
1900.16	1900.16	AMPS	550.16	AMPS
126.1967438	127.8969046	RADS	126.5054657	128.2717151
0.29	0.29	SEC	0.41	SEC
1787.66	1787.66	AMPS	437.66	AMPS
126.2309507	127.9365192	RADS	126.5208048	128.2926891
0.3	0.3	SEC	0.42	SEC
1675.16	1675.16	AMPS	325.16	AMPS
126.2636622	127.9746554	RADS	126.5344793	128.3120201
0.31	0.31	SEC	0.43	SEC
1562.66	1562.66	AMPS	212.66	AMPS
126.2948645	128.0112997	RADS	126.5464748	128.3296939
0.32	0.32	SEC	0.44	SEC
1450.16	1450.16	AMPS	100.16	AMPS
126.3245438	128.0464385	RADS	126.5567769	128.3456965
0.33	0.33	SEC	0.45	SEC
1337.66	1337.66	AMPS	-12.34	AMPS
126.352686	128.0800582	RADS	126.5653708	128.3600138
0.34	0.34	SEC		
1225.16	1225.16	AMPS		
	128.1121453	RADS		



Injection Porch

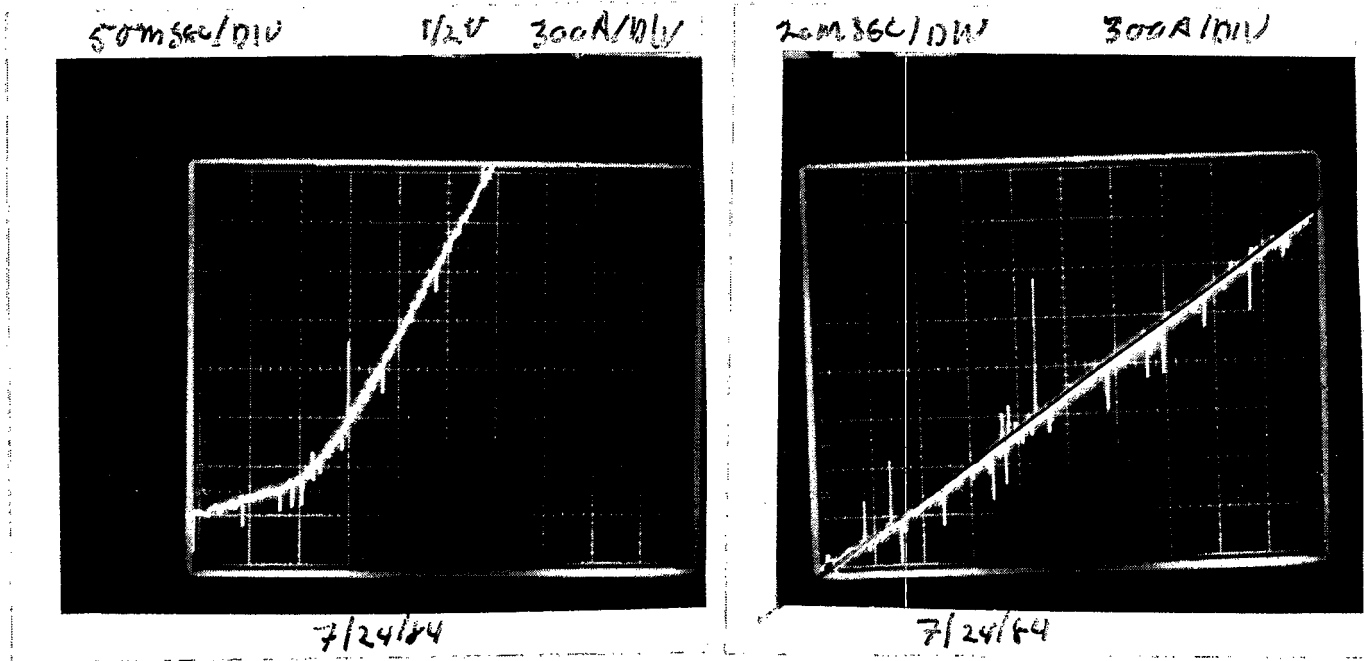


$$I = \frac{(120)(3)}{(0.02)(6.7)} = 2686.6t \approx 2687t \quad \left| \begin{array}{l} 0.11 \text{ sec} \\ 0 \end{array} \right.$$

$$I_{0.11 \text{ sec}} = 295.6 \text{ Amps}$$

FIGURE 1

Rectify



$$I = 295.6 + \frac{(300)(7)}{0.2} t^{-0.11} = 295.6 + 10,500 (t - 0.11)$$

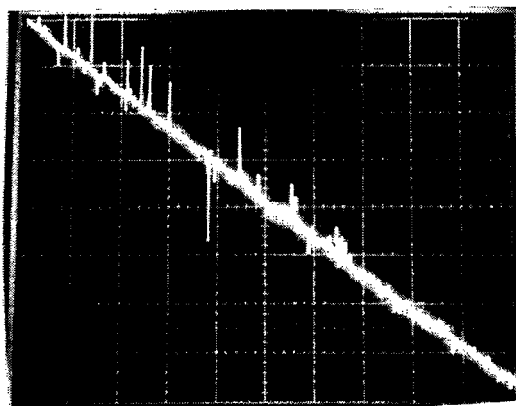
$$I = -860 + 10,500t \Big|_{0.11}^t$$

FIGURE 2

Invert

20 msec/Div

300 Amps/Div



$$I = I_F - \frac{(300)(7.5)}{0.2} = I_F - 11,250t$$

FIGURE 3